TECHNISCHE UNIVERSITEIT EINDHOVEN
FACULTEIT TECHNISCHE NATUURKUNDE
SUSTAINABLE ENERGY TECHNOLOGY

Examination Physical Transport Phenomena 3T320
1 November 2011, 9:00 u –12:00 u

(10 pnt) Problem 1.
Provide a short answer with arguments to the following questions.

(2 pnt) **a).** A flow is given by the following velocity field (vector):

$$\underline{v} = (v_x, v_y, v_z) = (-f \sin x \cos y \,, f \cos x \sin y, 2fxyt)$$

In the expression t represents the time and f is a constant.
Is this flow incompressible?
Calculate the value of $D\underline{v}/Dt$ for this flow.

(2 pnt) **b).** Show that $\nabla(\underline{v} \cdot \underline{w}) = \nabla\underline{v} \cdot \underline{w} + \nabla\underline{w} \cdot \underline{v}$, where $\underline{v}$ and $\underline{w}$ are position dependent vectors.

(2 pnt) **c).** An incompressible fluid with density ρ is flowing through a square pipe with sides D and a length L, shown in cross section in figure 1 on the left. The mass flow rate is W, and the pipe is installed horizontally. Now we like to replace this pipe by a circular pipe with diameter d and the same tube length L, see figure 1. The flow in the pipes is turbulent, developed, and can be described with a friction factor f given by the following Reynolds number dependency: $= \frac{1/10}{Re^{1/4}}$. The hydraulic radius is defined as, $R_h = \frac{pipe\ cross\ section}{wetted\ perimeter}$, and the Reynolds number is defined by $Re = \frac{4R_h\langle v\rangle\rho}{\mu}$. Here $<v>$ is the average velocity in the pipe, and μ is the dynamic viscosity.
State and use the correct balance equation to find an expression for the pressure drop over both types of pipes. Find the ratio d/D in case the mass flow rates through both systems are equal at the same pressure difference.

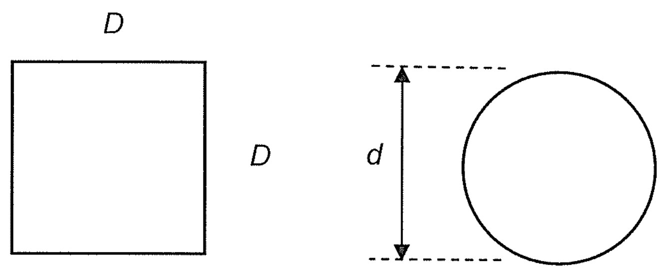

Fig. 1 Two pipes, one of square, and one with a circular cross section.

(2 pnt) **d).** Compute the solar constant of the planet Mars. The following data are given: Constant of Stefan-Boltzmann, σ=5.667·10^{-8} W/(m^{2}K^4); temperature of the Sun T_s=5762 K; diameter of the sun D_s=1.384·10^6 km; diameter of Mars, D_m=6775 km; distance between Sun and Mars, r_{12}= 228·10^6 km.
Derive/make a steady-state radiant energy balance for the surface of Mars that is facing the Sun, and regarding Mars as a flat circular disk facing the sun. Assume that only incoming radiation from the Sun, no convection, no conduction, and no reflections are taking place.
Estimate the surface temperature of Mars using the radiant energy balance.

(1 pnt) **e).** A person is standing in a large room, and his outside (skin) temperature is 308 degrees Kelvin. The temperature of the walls of the room is 285 degrees Kelvin. The emissivity of the person's skin is 0.90, and the room can be considered as a black body radiator. The surface area of the person is assumed to be 1.5 m^2 and σ=5.667·10^{-8} W/(m^{2}K^4).
What is the net heat loss via radiation by this person to the room?

(1 pnt) **f).** In figure 2 a one-dimensional wall is shown, and plotted is the thermal conduction coefficient as a function of the wall coordinate x. For the first 1/3 of the wall the thermal conduction coefficient is constant with value k_0, for the next section $1/3<x<2/3$ it rises linearly to a value $2k_0$, in the last section it is constant and has a value $2k_0$. At $x=0$ the temperature is T_1 and at $x=1$ the temperature is T_2. We assume that $T_1<T_2$. Take over the graph on your answering paper, and draw also the temperature in the wall (along y-axis) as a function of x (use another ballpoint or pencil colour if possible). Motivate why the behaviour is as you draw it. No calculations are asked, only some insight is needed.

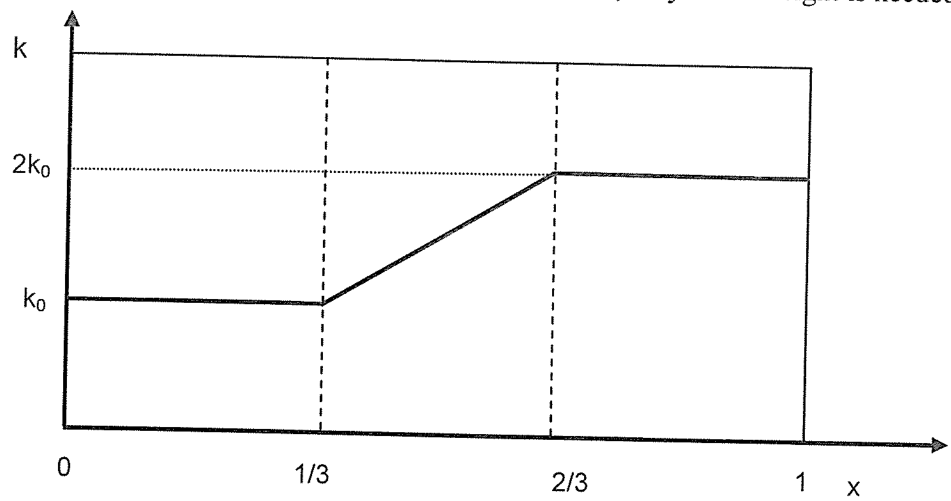

Fig. 2 Thermal conduction coefficient in a wall as a function of x.

(10 pnt) Problem 2

A long plan parallel flow channel is shown in figure 3 with height h, length L, and width (in z-direction) B. The walls of the channel are made of a porous material. The channel is filled with an incompressible Newtonian fluid with constant density ρ, and dynamic viscosity μ, and $\mu = \rho\nu$ where ν is the kinematic viscosity. There is a fixed pressure difference Δp between $x=0$ and $x=L$. The flow in the x-direction is steady and developed, and in the x-direction the no-slip condition holds on the walls. At the lower wall, additional fluid is homogeneously injected in the y-direction through the porous wall, and the same amount is extracted at the upper wall. The injected fluid enters and leaves the channel with velocity V in the y-direction. Gravity effects can be neglected. The flow is two-dimensional, so the z-component does not play a role in this problem. The velocities in the x, y and z-direction are named v_x, v_y, and v_z respectively.

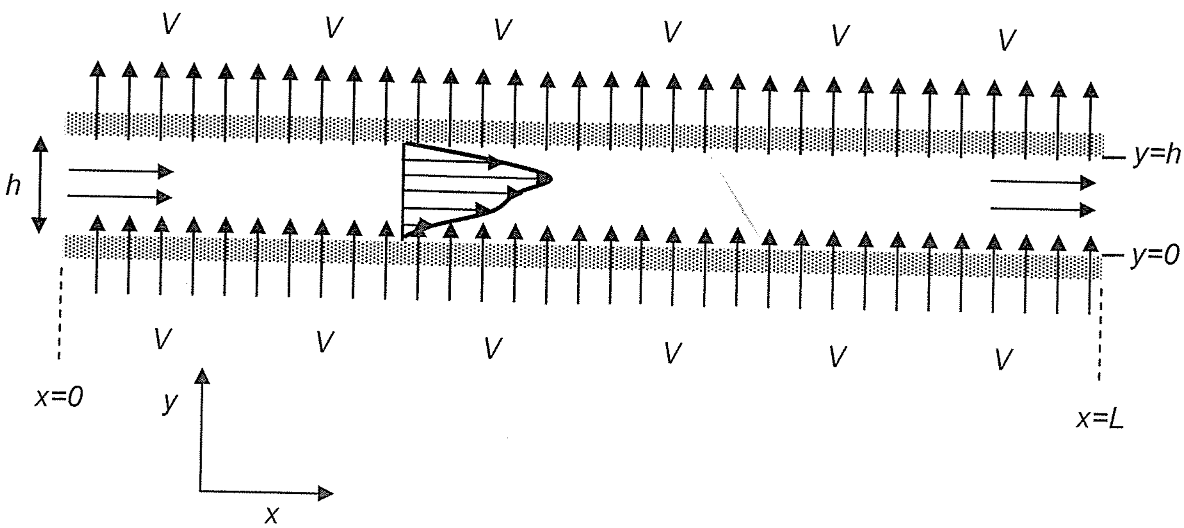

Figure 3 Parallel flow channel with porous walls.

(1 pnt) **a).** Formulate the boundary conditions for the horizontal (x) velocity component v_x, and the vertical (y) velocity component v_y.

(1 pnt) **b).** Proof by using the differential form of the continuity equation, that the vertical velocity component v_y is constant with a value V in the channel.

(1 pnt) **c).** Show that the x-component of the Navier-Stokes equation reduces to the following form, and motivate clearly why some terms can be neglected:

$$\nu \frac{d^2 v_x}{dy^2} - V \frac{dv_x}{dy} = \frac{1}{\rho}\frac{dp}{dx}$$

(1 pnt) **d).** Show that the particular solution of this differential equation is given by,

$$v_{xp} = \alpha y\,,$$

and determine the value of α.

(2 pnt) **e).** In order to solve the homogeneous part of the differential equation (in case the right hand side of the diff. eq. is zero) we introduce $f(y) = \frac{dv_x}{dy}$. Show that the solution of the homogeneous part of the differential equation is given by $f(y) = Ae^{(V/\nu)y}$, where A is a constant to be determined.

(3 pnt) **f).** Use the result of e) for $f(y)$ to find the solution for $v_x(y)$. Add the particular solution of answer d). to find the full solution of the problem. Apply the boundary conditions to find all integration constants.

Under the condition that $V \ll \nu/h$ it is possible to approximate the result of answer e). by a third order polynomial:

$$v_x(y) = y(y-h)\frac{\Delta p}{2\mu L}\left[1 + \frac{V}{6\nu}(2y-h)\right]$$

(1 pnt) **g).** Use the above polynomial for $v_x(y)$ to determine the mass flux through the channel.

(10 pnt) Problem 3.

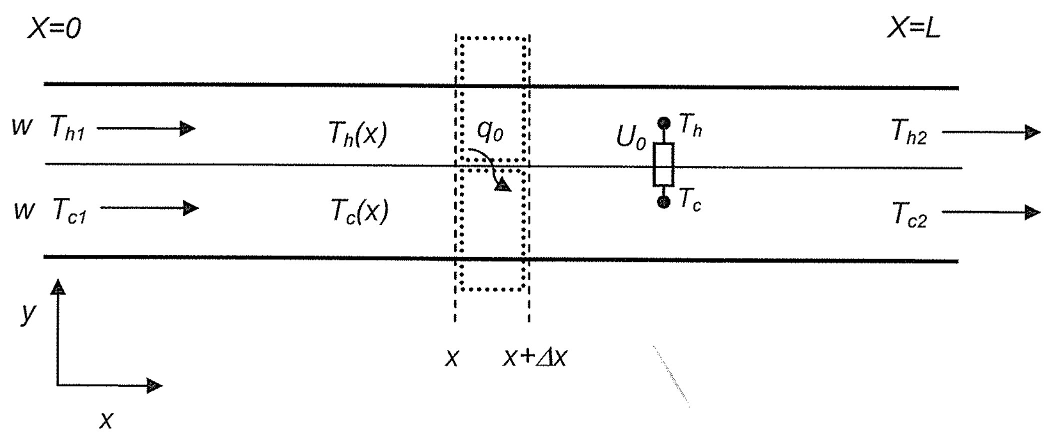

Fig. 4 Schematic drawing of the parallel flow heat exchanger.

A parallel co-flow heat exchanger is shown in figure 4. It consists of two flat parallel channels that are separated by a thin wall through which heat transfer takes place. The heat exchanger is exactly symmetrically designed, and both flow channels have the same geometry. The length of the heat exchanger is L and the width in the z-direction (normal to the paper) is W_0 . The wall separation distance of the flow channels can be indicated as d, but this is not a relevant parameter in the exercise. The hot and cold size inlet average temperatures are known and given as T_{h1} and T_{c1}. The exit (unknown) temperatures are T_{h2} and T_{c2}. The fluid is an incompressible fluid, with density ρ and is the same in both the hot and cold flow channel. Furthermore the mass flux given as w of both flows is equal. We assume that the flow is steady, fully developed, and that an effective total heat transfer coefficient U_0 takes place from fluid to fluid over the wall, see figure. This heat transfer is defined as: $U_0 = \frac{q_0}{T_h - T_c}$, where q_0 is the heat flux (in W/m^2), and $T_c(x)$ and $T_h(x)$ are the local average (x-dependent) temperatures. Furthermore the enthalpy per unit of mass can be approximated by: $\hat{H} = \hat{C}_p T$. There is no further heat exchange with the outside environment, the kinetic energy in the fluid can be neglected, and also gravity effects can be neglected, and there is no application of external shaft work to the heat exchanger.

(1 pnt) **a).** Formulate in a formula the steady state macroscopic energy balance, and explain what the terms mean. Indicate which and why some terms can be neglected for the analysis of the heat exchanger.

(1 pnt) **b).** We define now for the total lost or gained heat flow (in W) over the complete hot, or cold flow channel the symbols Q_h and Q_c. What is the relationship between Q_h and Q_c? Apply the macroscopic energy balance of a) to the complete hot flow channel to derive an expression for Q_h. Repeat this procedure for the cold flow channel.

(1 pnt) **c).** Now we focus on the dashed zones as indicated in figure 4 at x to $x+\Delta x$. Express the effective exchanged heat flow ΔQ for this segment of the hot flow channel in terms of the local temperatures at x and $x+\Delta x$. Do the same for the cold flow channel.

(2 pnt) **d).** Apply the macroscopic energy balance to the dashed zone from x to $x+\Delta x$ separately for the hot flow channel, and the cold flow channel, as indicated by the dashed boxes in the figure, and show that,

$$\frac{dT_h}{T_c - T_h} = \alpha dx$$
$$\frac{dT_c}{T_c - T_h} = -\alpha dx$$

and specify the value of α.

(2 pnt) **e).** Find the solution for T_h-T_c as a function of x also using the inlet boundary condition.

(2 pnt) **f).** Use the differential equation for T_h to find the expression for T_h as a function of x.

(1 pnt) **g).** Compute Q_h the net heat lost in the hot flow channel.

END EXAM