

**TECHNISCHE UNIVERSITEIT EINDHOVEN**  
**FACULTEIT TECHNISCHE NATUURKUNDE**  
**GROEP TRANSPORTFYSICA**

Examination "Physical Transport Phenomena" 3T320  
 3 November 2009, 9:00 u – 12:00 u

**(11 pnt) Problem 1.**

Provide a short answer with arguments to the following questions.

- (2 pnt) a). What is the meaning of the term  $\underline{\tau} : \nabla \underline{v}$ ?

In which equation does this term appear?

Consider the incompressible flow field  $(v_x, v_y, v_z) = (-\frac{1}{2}bx, -\frac{1}{2}by, bz)$ . Compute  $\underline{\tau} : \nabla \underline{v}$  for this flow field.

- (2 pnt) b). Consider in the Cartesian coordinate system  $(x, y, z)$  the time dependent velocity field

$$\underline{v} = (v_x, v_y, v_z) = \left( -\frac{1}{2}x \left( \frac{1}{\sqrt{y}} + \frac{1}{\sqrt{z}} \right) \frac{1}{\sqrt{t}}, \sqrt{\frac{y}{t}}, \sqrt{\frac{z}{t}} \right).$$

In which  $t$  is the time.

Is this flow incompressible? Calculate  $D\underline{v}/Dt$ .

- (2 pnt) c). Specify the conditions for the validity of Bernoulli's law in the form:

$$p + \frac{1}{2} \rho V^2 + \rho gh = \text{constant}$$

Consider flow around a circular cylinder normal to its axis as shown in figure 1 below.

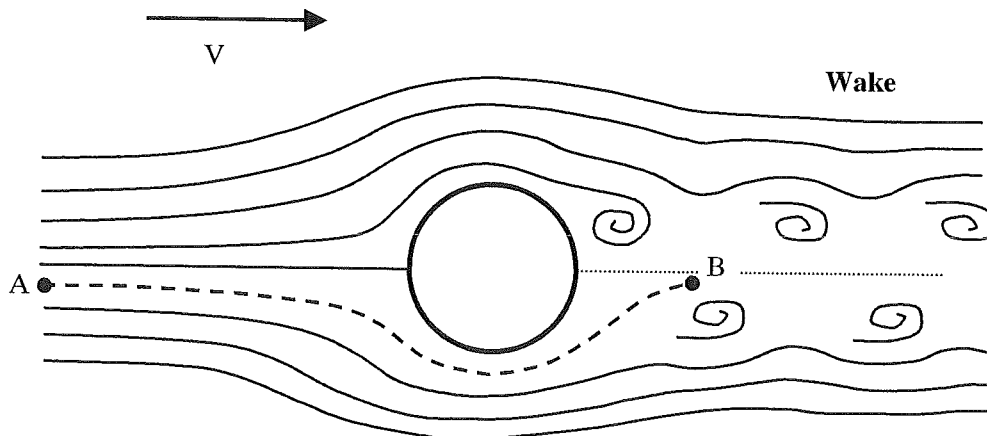


Fig.1 Flow around a circular cylinder.

Why can we not apply Bernoulli from point A upstream of the cylinder to a point B downstream in the wake. Which assumption fails?

- (2 pnt) d). Compute the solar constant of the planet Mars. The following data are given: Constant of Stefan-Boltzmann,  $\sigma = 5.667 \cdot 10^{-8} \text{ W}/(\text{m}^2 \text{K}^4)$ ; temperature of the Sun  $T_s = 5762 \text{ K}$ ; diameter of the sun  $D_s = 1.384 \cdot 10^6 \text{ km}$ ; diameter of Mars,  $D_m = 6775 \text{ km}$ ; distance between Sun and Mars,  $r_{12} = 228 \cdot 10^6 \text{ km}$ .

Derive/make a steady-state radiant energy balance for the surface of Mars that is facing the Sun, and regarding Mars as a flat gray disk. Assume that only incoming radiation from the Sun and no convection, no conduction, nor any reflections are taking place. Estimate the surface temperature of Mars using the radiant energy balance. What will happen more towards the poles of Mars?

- (3 pnt) e). Two parallel double glazing window panes with temperatures  $T_1$  and  $T_2$  are displaced at a distance  $d$  from each other and they exchange heat via thermal radiation, and thermal conduction via air with thermal conductivity  $k$ . The heat transfer by radiation is given by:

$$q_r = \sigma(T_1^4 - T_2^4).$$

Determine a formula for the temperature difference  $T_1 - T_2$  for which the heat transfer by radiation equals heat transfer via conduction. *Hint: use the expression that  $T_2 = T_1 + \Delta T$  and assume that  $\Delta T/T_1 < 1$  and linearize to first order.*

Now  $\sigma = 5.667 \cdot 10^{-8} \text{ W/(m}^2\text{K}^4)$  and  $k = 0.024 \text{ W/(mK)}$ ,  $T_1 = 273 \text{ K}$ , and  $d = 0.005 \text{ m}$ . Calculate the temperature difference between the two window panes.

Are there circumstances under which thermal radiation is always dominant under arbitrary conditions?

## (12 pnt) Problem 2

When a container filled with liquid is emptied a layer of fluid clings to the inside walls of the container as a thin film left behind on the wall. This unsteady process is shown schematically in cross-section in figure 2a below.

Before attempting to find a solution for a withdrawing fluid film on a wall we like you first to focus on figure 2b. Here is shown a fluid film that is continuously supplied as a thin layer higher-up (not indicated) on a wall, and flows downwards due to gravity forces as indicated with  $g_z$ . The thickness of the film is  $\delta$  and the global velocity profile of the speed inside the film is indicated. In the drawing the  $x, y, z$ -axis system is shown. The origin is on the wall. A control volume with ribs of length  $L$ ,  $\Delta x$  and  $W$  is also indicated. It is assumed that the film thickness  $\delta$  is so thin that the velocity in the film is only in the direction of  $z$  and is only a function of  $x$ , so  $\underline{v} = (v_x, v_y, v_z) = (0, 0, v_z(x))$ . It is assumed that the flow is steady, stable and developed. We assume that at the liquid film-gas interface all stress components,  $\tau_{xx}$ ,  $\tau_{xy}$ ,  $\tau_{xz}$  are zero. Furthermore the pressure on the outside and inside of the film is constant and has a value  $p_0$ . The fluid is incompressible.

- (1 pnt) a). Motivate why the thickness  $\delta$  of the film is constant when moving over  $z$ .
- (1 pnt) b). Set-up and evaluate the momentum balance equation in the  $z$ -direction for the control volume element shown in the drawing, using the total momentum tensor  $\Phi_{ij}$  and relevant forces and show that:

$$\frac{\partial \Phi_{xz}}{\partial x} - \frac{\Phi_{zz}|_{z=z_0} - \Phi_{zz}|_{z=z_0+L}}{L} = \rho g_z.$$

Please motivate clearly in your derivation why you neglect certain terms.

- (1 pnt) c). Motivate why at the interface between the film and air  $\tau_{xz} = 0$ . How is this condition expressed in terms of  $v_z$  at the interface for a Newtonian fluid? Please state the second boundary condition.

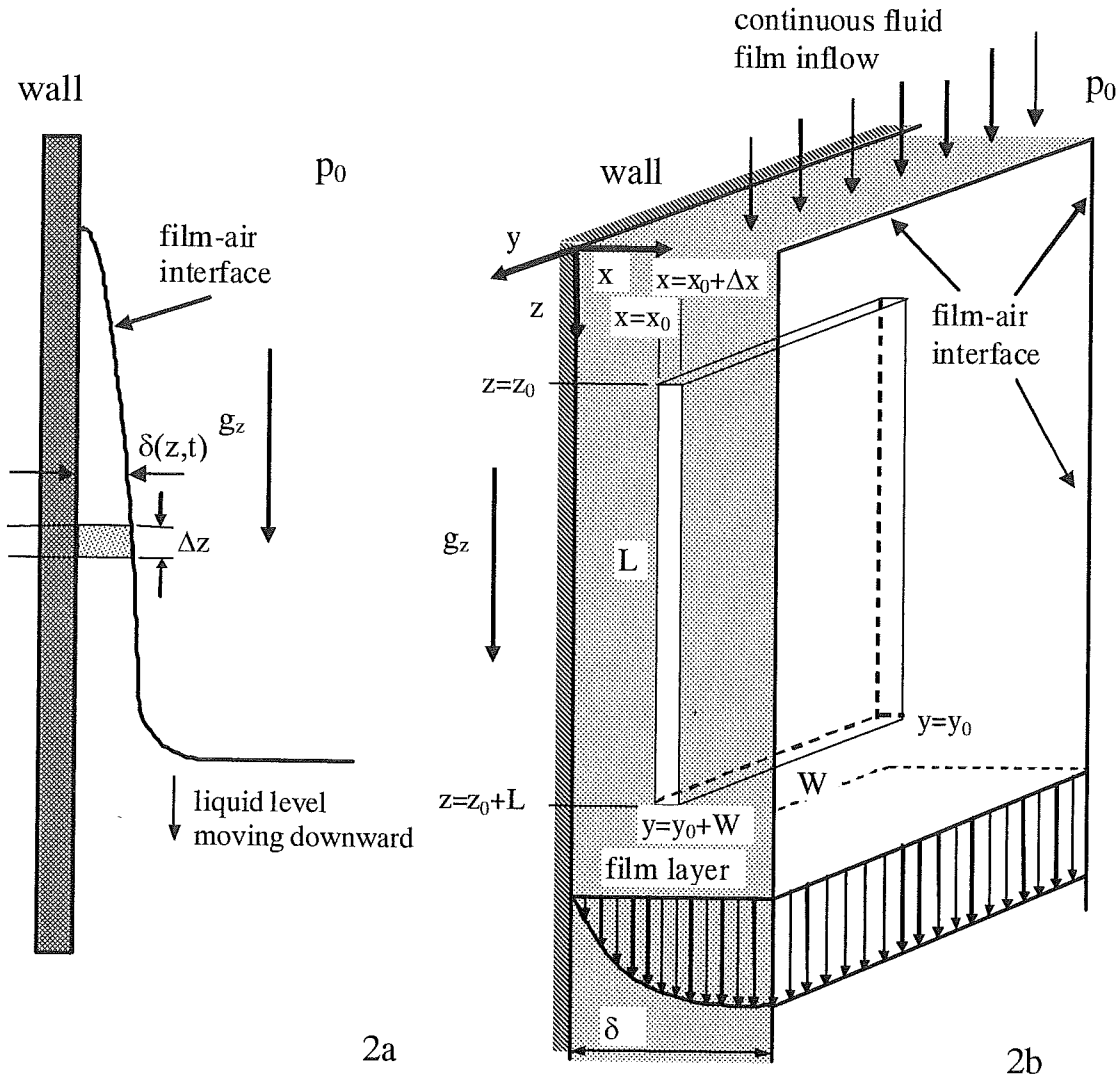


Fig. 2a

Fluid film withdrawing from a wall.

Fig. 2b

Continuous flow of a film downwards on a wall

- (1 pt) **d).** Evaluate the momentum equation in b) further by using the expressions for the shear stresses in case of a fluid for which Newton's law of viscosity holds and prove that:

$$\frac{\partial^2 v_z}{\partial x^2} = -\frac{\rho g_z}{\mu}.$$

- (1 pt) **e).** Derive the velocity profile  $v_z(x)$  in the fluid film using the boundary conditions.

- (2 pt) **f).** Show that the average velocity in the film is given by:

$$\langle v_z \rangle = \frac{\rho g_z \delta^2}{3\mu}.$$

We return now to the withdrawing fluid film as shown earlier in fig. 2a. Fig. 2a shows in a snapshot as if a photo were made instantly the film profile. Now as there is no continuous supply of fluid from above the thickness of the film will change over time and height. So the thickness of the film depends also on time and the process is unsteady. So now  $\delta = \delta(z, t)$ .

- (2 pnt) g). Make an unsteady-state continuity (mass) balance for the indicated fluid film element in figure 2a in terms of the average velocity  $\langle v_z \rangle$  and  $\delta(z, t)$  and show that:

$$\frac{\partial \delta}{\partial t} = -\frac{\partial(\delta \langle v_z \rangle)}{\partial z}.$$

To find an expression for the time versus  $z$  dependency of the film thickness in the unsteady withdrawing film flow we assume now that the solution for the steady flow and the relationship between  $\delta$  and  $\langle v_z \rangle$  found in the first part of this exercise will also hold for the unsteady film flow. This is called a quasisteady assumption.

- (2 pnt) h). Use the expression found in 2f). together with the result in 2g). to find a differential equation for the film thickness  $\delta$ .

Show that the expression,

$$\delta(z, t) = \frac{1}{\sqrt{\frac{\rho g_z}{\mu}}} \sqrt{\frac{z}{t}},$$

satisfies the differential equation.

### (7 pnt) Problem 3.

Consider a very long electrical wire (length  $L$ ) of circular cross section (radius  $a$ ), see figure 3. The wire has an electrical resistance  $R$ . An electrical current  $I$  passing through the wire produces a uniform dissipation of heat  $\phi$ .

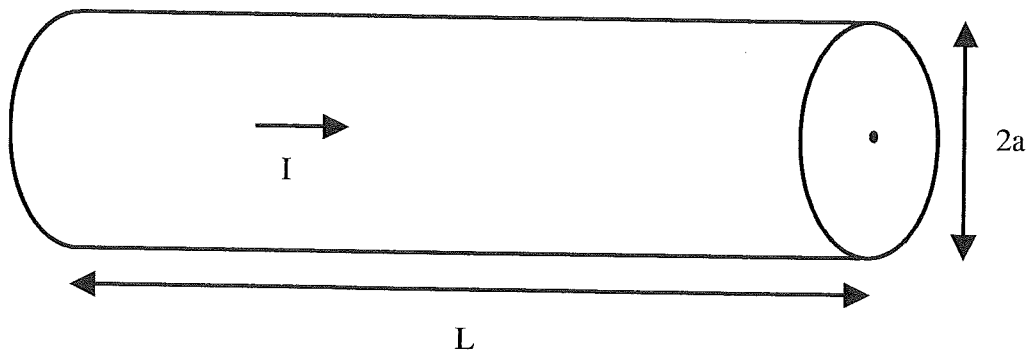


Fig. 3 Cylindrical wire carrying a current

- (1 pnt) a). Show from an overall energy conservation balance of the wire that the steady heat flux  $q_r(a)$  from the surface of the wire to its surroundings is given by:  $q_r(a) = \frac{RI^2}{2\pi aL}$ .
- (1 pnt) b). We assume that the heat exchange between the wire and its surroundings is dominated only by black body radiation:  $q_r(a) = \sigma(T_w^4 - T_\infty^4)$  where  $\sigma$  is a constant,  $T_w = T(a)$  and  $T_\infty$  is the temperature of the surroundings. Calculate  $T_w = T(a)$  as a function of the problem parameters.
- (1 pnt) c). One can interpret the electrical current heat source in terms of a heat source density  $\phi$  (so per unit of volume). Compute  $\phi$ .

- (2 pnt) d). Using an energy balance applied to a thin cylindrical shell, show that within the wire:  
 $\frac{1}{r} \frac{d(rq_r)}{dr} = \phi$  where  $r$  is the distance to the axis of the wire.  
Calculate  $q_r$  as a function of  $r$ .
- (2 pnt) e). Assuming a constant heat conduction coefficient  $k$  of the wire, calculate the temperature distribution within the wire  $T(r)$ .

**END EXAM**