3F011 Written Examination

6 November 2013

9:00 – 12:00

READ ME FIRST

This exam is 'back of the envelope'. So, only rough estimates are required, typically order of magnitude. But it is important that you indicate how you come to your approximate numbers, i.e. what assumptions you make and where you use rounded numbers etc. Mostly sound reasoning is all that is required.

However: don't mistake rounding off for bad arithmetic!

All information needed is given in the formula sheet or in the problems themselves.

We expect you to know a few basic formulas and to be able to derive others when you need them.

You are not allowed to use a calculator, books, lecture notes, etc . This course is about solving problems with your basic science toolbox.

1. Put your name on each sheet of paper.
2. Clearly indicate on the paper which problem it answers.
   Mark sub-questions with 1a, 1b etc.
   (Not just a, b – I will not search for missing bits of a problem/answer!)
3. The questions are in English. You can answer in Dutch if you wish.
4. If you use symbols, ALWAYS define and/or explain them.
5. There are 5 problems, each with sub-questions. Sometimes a sub-question itself contains several small questions. Make sure to answer all.
6. The points that can be earned with each question are indicated behind each question as a number between brackets: (2). Not all questions have equal weight – spent your time wisely.
7. Most questions that start with 'describe' can be answered in 1 or 2 sentences. Please do not write essays! (But if you do, write neatly).

Formulas, constants and other useful stuff

Fundamental constants:

Electron mass:	$m_e = 9.1 \times 10^{-31}$ kg
Proton mass:	$m_p = 1.7 \times 10^{-27}$ kg
Electron charge:	$e = 1.6 \times 10^{-19}$ C
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}$ N/A^2
Boltzmann's constant	$k_B = 1.4\ 10^{23}$ J/K
Planck's constant	$h = 6.6\ 10^{-34}$ J.s
Avogadro's number	$N_A = 6.0 \times 10^{23}$

Energy from fusion reactions: D-T: 17.6 MeV D-D: 3.6 MeV

Planck black body radiation law

$I(\nu,T) \propto \nu^3 \{\exp[h\nu/kT] - 1\}^{-1}$ (I= intensity, ν=frequency, h=Planck's constant, T=temperature)

The total power per unit area emitted in 2π steradian by a black body radiator is given by:

$P_{BB} = 6 \times 10^{-8} T^4$ Wm^{-2}K^{-4} (Stefan-Boltzmann)

Collision times

$\tau_e = 3.\ 10^{-10}\ T_e^{3/2}\ n_{20}^{-1}$ s with T_e in eV and n_{20} in 10^{20} m^{-3}.

$\tau_i = (m_i/m_e)^{1/2}\ \tau_e = 10^{-8}\ T_e^{3/2}\ n_{20}^{-1}$ s (for hydrogen)

Mean free paths (λ)

Parallel to B: $\lambda_e = v_{th,e}\ \tau_e = \lambda_i = 10^{-4}\ T^2/n_{20}$

Perpendicular to B: gyro-radius $\rho = mv_\perp/qB$

Conductivities

$\chi_{//,e} = (\lambda_e)^2/\tau_e = (v_{th,e})^2\tau_e = [16\ 10^{10}\ T_e]\ [3.\ 10^{-10}\ T_e^{3/2}\ n_{20}^{-1}] \approx 50\ T_e^{5/2}\ n_{20}^{-1}$ m^2/s (T_e in eV, n_{20} in 10^{20} m^{-3})

$\chi_{perp,e} = (\rho_{ce})^2/\tau_e = (v_{th,e}/\omega_{ce})^2/\tau_e$ ($\omega_{ce} = eB/m_e \rightarrow 2\pi.28$ GHz per Tesla)

(for ions: work out)

Material properties

Deuterium: atomic weight: 2 proton masses (A=2); ionization energy: 13.6 eV

Other stuff

$\beta = \langle p\rangle/\langle B^2/2\mu_0\rangle$	<.> denotes volume average, B is the total magnetic field β is the ratio of the total kinetic pressure (p) to the magnetic pressure.
β-limit: $\beta_{max} \propto I/aB$	with I the plasma current, a the minor radius
$q_{cyl} = a.B_{tor}/RB_{poloidal}$	cylindrical safety factor, a, R = minor, major radius

Wavelength H_α = 656 nm

1. Development of Fusion Energy. (20)

a. Before fusion can be rolled out as energy source several scientific and/or technical issues need to be resolved. Give (just mention) two major issues. **(2)**

b. Choose one of the two you mentioned under (a) and describe why it is a problem, and sketch what can be done about it. **(3)**

c. A few things you ought to know about ITER **(3)**:

- What is the goal of ITER in terms of the 'power multiplication factor' Q and pulse duration.
- When should these goals be achieved according to the present planning?
- What is the estimated cost of the experiment (construction, not operation)?

d. ITER will start by operating in hydrogen, then – after a year or so - move to operation in deuterium and finally go to deuterium-tritium (d-t) operation. Why not use d-t from the start? **(2)**

e. The mission of ITER is partly to confirm the predictions for confinement and performance. But in other areas ITER will enter entirely new territory. Give two areas where ITER explores uncharted waters and briefly indicate why this is new in ITER. **(2)**

f. Supposing that ITER is a success, and that fusion power from there on will follow the historical trends also shown by fission, wind and solar: doubling the installed power every 3 years, **(4)**

- When will we have 100 $GW_{electric}$ of fusion power? (3)
- Roughly what fraction of the total energy demand of the world is that? (1)

g. A fusion reactor will have a 'blanket'. **(3)**

- What is the purpose of the blanket? (2)
- Will ITER have a blanket? (1)

h. Some parts of the reactor will be activated by neutrons, and these must be replaced from time to time. How long – approximately – must the activated parts that have been removed be stored before the radiotoxicity is reduced to a level comparable to the ash of a coal plant? (per Joule energy produced). **(1)**

2. The Lawson criterion (15)

a. The Lawson criterion (in a simplified form) states: $n.\tau_E.T$ > constant. **(2)**

- What are the 3 quantities in this 'triple product'. (1)
  (where necessary specify whether they refer to ions or electrons)
- Give the definition of τ_E. (1)

b. Derive the Lawson criterion. (only the proportionalities, no constants asked in the derivation). **(3)**

c. The constant in the Lawson criterion has the dimension [Pa.s]. What are the approximate values for the product (nT) and τ_E aimed for in ITER. **(2)**

d. The kinetic pressure must be kept in check by the magnetic field: **(4)**

- How large is the magnetic pressure of a 6 Tesla field? (2)
- Which part of the reactor contains this pressure? (2)

e. How low should the pressure in the tokamak vessel be before you let in the hydrogen (or deuterium or tritium) gas that will be ionised to form the plasma? Give a value (order of magnitude) + a reasoning. **(4)**

3. Confinement in a tokamak reactor (15)

a. A (modified for the purpose) H-mode scaling law might read:

$$\tau_E \propto I^{0.9} B^{0.2} P^{-0.7} n^{0.4} R^{1.8}$$

Bring this into a form that uses as much as possible the dimensionless numbers q_{cyl}, β, and ε, while eliminating P. **(5)**

b. Suppose for technical or financial reasons it must decided that the ITER toroidal field magnets will be operated at a value of $B_{toroidal}$ that is 10% lower than the design value. What would the implication be for the power multiplication Q? (explain your result – there are more than one correct answers, the correctness depends on your reasoning and correct statement of assumptions). **(5)**

c. ITER and future reactors may have to operate at a plasma temperature much higher than the optimum (i.e. from the point of maximum Q) burn temperature. Why? **(2)**

d. For the fusion reaction, only the ions need to be hot. The electrons have no other function than to neutralize the plasma. However, if the electrons are hot they cause heat losses. Can we operate a burning plasma (like ITER) with 'cold' electrons (i.e. $T_e << T_i$)? Why or why not: motivate your answer. **(3)**

4. Classical and Neoclassical transport. (30)

a. We start with the simplest diffusive transport. **(3)**

- Describe the mechanism of classical, diffusive cross-field transport in a magnetised plasma. (2)
- Give the generic relation between the (test particle) diffusion coefficient and the mean free path and collision time of the particle. (1)

b. Which is the dominant heat loss channel in a tokamak, still assuming classical transport only: ion or electron heat conduction? Demonstrate your answer (assume that the ion and electron temperature are the same). **(4)**

c. Express the energy confinement time, τ_E, of a system (this is in fact not specific to a fusion reactor at all) in terms of the system size (a) and the thermal diffusion coefficient χ. **(2)**

d. Using this expression and that for the classical diffusion coefficient to find an expression for the confinement time of a tokamak reactor, should the transport have been classical. **(4)**

e. Consider a tokamak reactor with plasma parameters (temperature and density – hence fusion power density) like ITER, and also otherwise everything the same (shape, B-field) except the absolute size. This is represented by the minor radius a.

Estimate the minimum value of a at which this reactor would 'burn' (keep its plasma warm with its own fusion power production) if the transport is classical. **(5)**

f. Would the α-particles that are borne in the fusion reaction be confined in this device? If not, is this a problem? (present your arguments – you may score points even if you got the wrong size in e) **(3)**

g. Now consider the banana-orbit: **(3)**

- which effect causes its existence in a tokamak? (1)
- How does the banana width relate to the Larmor radius of the electron (2)

h. Consider an electron at a point of time when it is situated in the horizontal midplane, on the outboard side (=low field side) of a tokamak. Whether this electron is trapped in a banana orbit or not depends on its pitch angle θ and on the aspect ratio ϵ of its position. Explain – qualitatively – why θ and ϵ are the determining parameters. **(4)**

i. Describe qualitatively one of the following two: the Bootstrap current or the Ware Pinch. **(2)**

5. Heating and measuring the plasma temperature. (20)

In Neutral Beam Heating (NBI) high-energy neutral particles are injected into the plasma.

a. When we say 'high energy', to what should you relate this energy? **(1)**

b. In JET the energy of the NBI injected particles is typically 100 keV. Why isn't a much higher energy used? **(1)**

c. For ITER injection energies of 0.5 or 1.0 MeV are foreseen. Which of the two energies is more effective (at the same beam power) if you want to spin up the plasma? Explain your answer. **(2)**

d. Which population does NBI heat most: the ions or the electrons? (support your answer with a reasoning). **(2)**

You can determine the ion temperature from a spectroscopic measurement of the H_α emission line.

j. Why is it necessary to have NBI (either the heating beam or a dedicated diagnostic beam) for this measurement? **(2)**

For the electron temperature (T_e), you could use Electron Cyclotron Emission spectroscopy.

k. Show that for an optically thick plasma the ECE-intensity is proportional to T_e. **(4)**

Suppose there is a small population of supra-thermal electrons in the plasma of JET.

l. Which would give the more reliable measurement of T_e: an ECE antenna that looks into the plasma from the high field side or from the low field side? Explain your answer. **(4)**

You want to determine the electron heat diffusivity with a perturbative measurement. Therefore you want to somehow modulate T_e and measure the propagation of the perturbation with ECE.

m. Would modulation of the NBI be a suitable way of perturbing T_e? Why or why not? **(4)**