TECHNISCHE UNIVERSITEIT EINDHOVEN
FACULTEIT TECHNISCHE NATUURKUNDE
SUSTAINABLE ENERGY TECHNOLOGY

Examination "Physical Transport Phenomena" 3T320
24 January 2012, 9:00 u –12:00 u

(8 pnt) **Problem 1**

Provide a short answer with arguments to the following questions.

(2 pnt) **a).** A velocity field is said to be irrotational if $\nabla \times \underline{v} = 0$. Which of the following fields are irrotational? Field 1: $v_x=by$, $v_y=0$, $v_z=0$. Field 2: $v_x=-by$, $v_y=bx$, $v_z=0$.
Compute also $\nabla\underline{v}$ for both velocity fields.

(2 pnt) **b).** In figure 1 the thermal conduction coefficient of a wall is plotted as a function of the wall coordinate x. For the first 1/3 of the wall the thermal conduction coefficient is constant with value k_0, for the next section $1/3<x<2/3$ it has a value $4k_0$, in the last section a value $2k_0$. At $x=0$ the temperature is T_1 and at $x=1$ the temperature is T_2. We assume that $T_1<T_2$. Take over the graph on your answering paper, and draw also the temperature in the wall (along y-axis) as a function of x (use another ballpoint or pencil colour if possible). Motivate your answer. No calculations are asked, only some insight is needed.

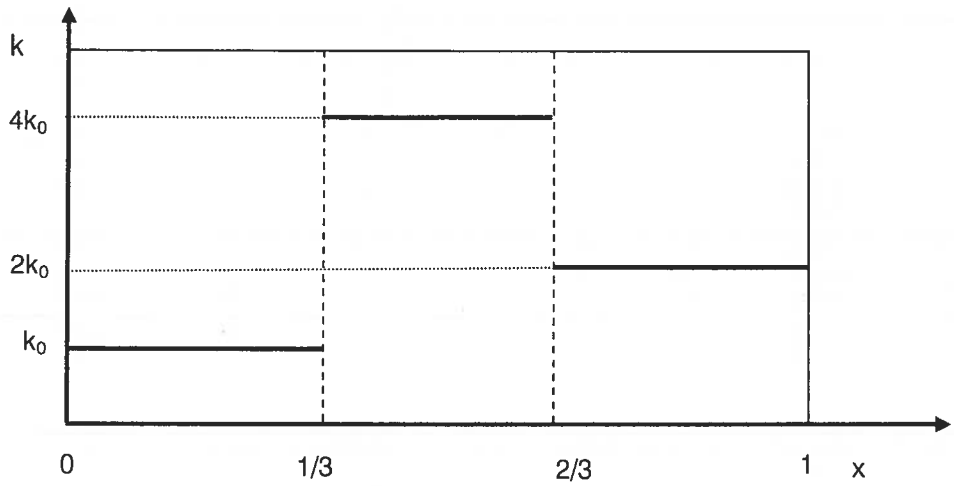


Fig. 1 Thermal conduction coefficient in a wall as a function of x.

(2 pnt) **c).** In figure 2 two situations are drawn of a wall (1) with radiation heater (5) mounted parallel. On the wall a thin sheet of insulation material (3) with a thickness d is glued. The insulation material has two different top-coatings. One coating (2) has an emissivity $\varepsilon_2 = 1$, the other coating (4) has emissivity $\varepsilon_4 << 1$. In case a) the coating with emissivity $\varepsilon_2=1$ is glued on the wall and the other coating is made facing the heater, while in case b) the coating with $\varepsilon_2=1$ is made facing the heater, and the coating with low emissivity is glued on the wall. In the insulation layer there is only heat conduction with a certain thermal conduction coefficient k. In between the radiator, and the top plane we assume that only thermal radiation occurs. The emissivity of the heater is $\varepsilon_5=1$.

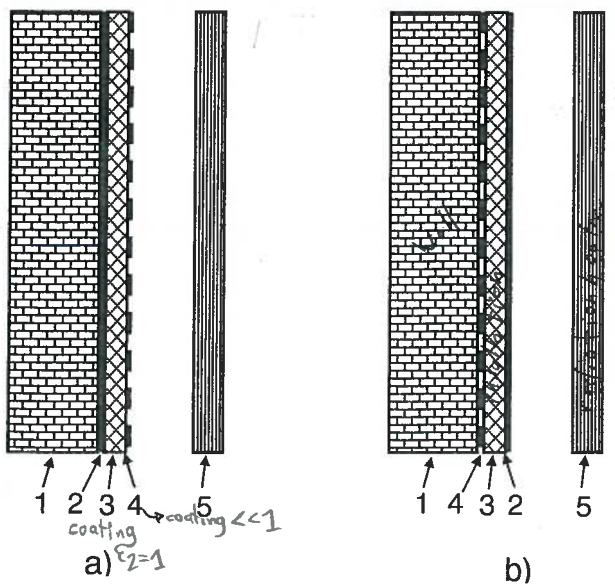


Fig. 2 Radiator on wall.

Determine which case a) or b) gives lower energy loss to the wall. Motivate your answer. For the radiation heat transfer you may use the formulas for flat parallel gray surfaces with a view factor of 1. You may assume that the temperature of the whole wall as well as the glued side of the insulator has a value T_1 and the temperature of the radiator is T_5; it is helpful to define the temperature of the radiation side of the insulator as T_m. In order to make the analysis simple you may assume for the radiation equation that: $T_5^4-T_m^4=4T_5^3(T_5-T_m)$.

(2 pnt) **d).** A student likes to test the Poiseuille equation for laminar flow in a circular tube. In order to do this he uses a reservoir and connects a tube to the outflow opening of the reservoir (see figure 3). The experiments are performed in a laboratory where the atmospheric pressure has a constant value (p_0). He/she uses water as liquid, and makes sure that the water level (ε) in the upper reservoir is constant via a supply system (not shown). The length of the tube is ℓ the diameter d. The reservoir is designed in such a way that the height ε is negligibly small ($\varepsilon<<\ell$), so that no additional hydrostatic effects occur. A second reservoir is placed below the outflow opening to collect the liquid mass that the student will measure after the experiment is made. The outflow opening is above the water level of the lower reservoir, so that hydrostatic effects do not play a role at the outflow side. The student carefully measures the mass of water that is collected over a specific time. In this way the mass flow rate (w) is known.
By using many different tubes with all the same length but varying diameter, the student wants to verify the laminar relationship between mass flow rate and the tube diameter. The students knows the density (ρ), dynamic viscosity (μ) and/or kinematic viscosity (ν), and the gravitation (g). Capillary effects are neglected.

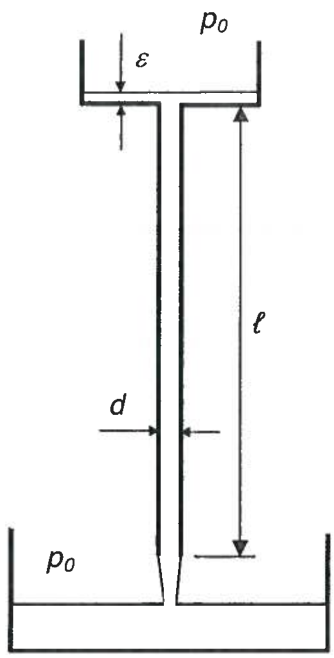


Fig. 3 Set-up for flow test

Write down the relation that describes the mass flow rate (w) in terms of the other parameters (d, g, ν, μ, ρ, ℓ) as far as these play a role in the problem.
Is the length (ℓ) a relevant parameter in this problem?
Taking into account that at a Reynolds number Re=2300 turbulent flow sets in, what is the maximum diameter that can be used in this laminar flow experiment? Use the following data: $\mu=\nu\rho$ and $g=10$ m/s^2, $\mu=0.001$ Pas, $\rho=1000$ kg/m^3, $\ell=1$ m.
What sort of problems will the student encounter in the experiment and analysis?

112
373
608

(11 pnt) Problem 2

Out of a cylindrical container, a very viscous incompressible, and Newtonian liquid is flowing through a circular hole with diameter $d=2R_0$. The fluid enters a thin slit between the container upper wall, and a lid that is fixed on top of the container. The radius of the container, and lid is R_1. The pressure in the container is p_0 and the pressure outside the container is p_1. The slit width is constant and has a value of h. We assume that $h<<R_1$. The flow velocity in the slit is assumed to be steady, and has only a radial component (so $v_\theta = v_z = 0$). Gravitation is neglected. There are no inlet effects at $r=R_0$. We use a cylindrical (r, θ, z) coordinate system, with velocities (v_r, v_θ, v_z), see fig. 4.

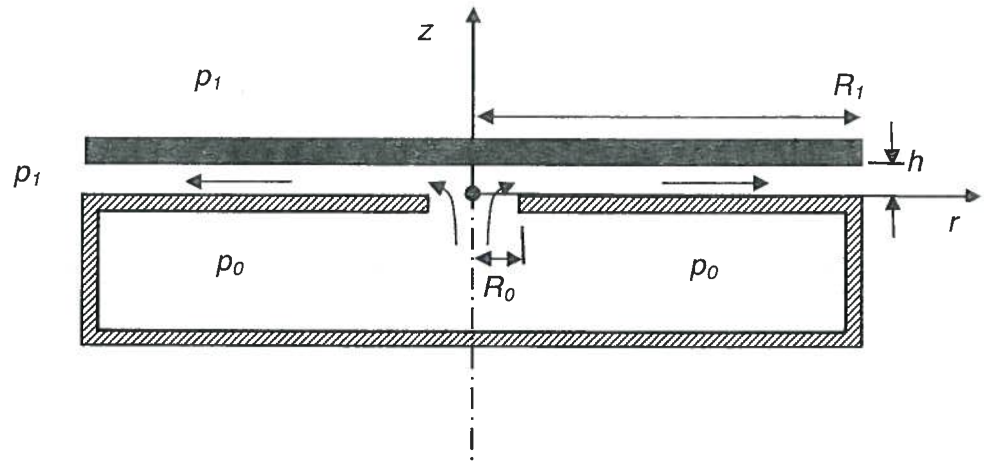


Figure 4 *Flow through radial slit.*

(1 pnt) **a).** Show with the continuity equation in cylindrical coordinates, that the radial velocity component v_r of the slit flow can be written as $v_r=c(z)/r$, where $c(z)$ is a function of the z-coordinate.

(1 pnt) **b).** Write down the r-component of the Navier-Stokes equation, and describe in words the terms (how they are named). Mention clearly which and why some terms are zero.

(1 pnt) **c).** In the exercise it is mentioned that the viscosity is very large, or in other words the Reynolds number (Re) will be very small so the flow is viscosity dominated. Show that by also neglecting the inertia terms in answer b), the equation can be reduced to: $\frac{\partial p}{\partial r} = \mu \frac{\partial^2 v_r}{\partial z^2}$.

(1 pnt) **d).** Use now the axial (z)-component of the Navier-Stokes equation, and simplify this by neglecting terms to proof that the pressure in the radial slit is only dependent of r. So $p=f(r)$.

(1 pnt) **e).** As p is only r-dependent, so is also $\partial p/\partial r$, or in a formula $\partial p/\partial r=g(r)$. Combine this with the answer of c). to show that:

$$v_r(r,z) = \frac{1}{2}\frac{g(r)}{\mu}z^2 + k(r)z + l(r).$$

Here $k(r)$ and $l(r)$ are functions to be determined.

(2 pnt) **f).** Write down the boundary conditions for v_r and write down the boundary conditions for p.

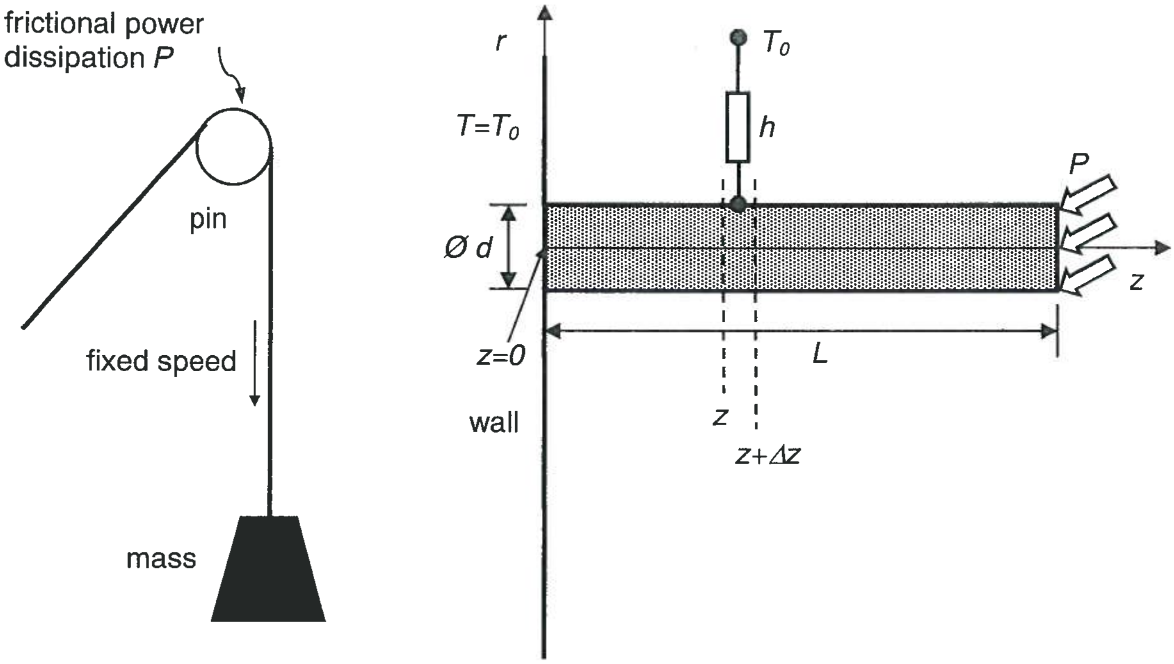
(3 pnt) **g).** Use the boundary conditions for v_r and the result of a). to proof that:

$$v_r(r,z) = \frac{C}{2\mu r}(z^2 - hz)$$

(1 pnt) **h).** Find the pressure as a function of the radius r and determine the value of C.

(11 pnt) Problem 3

A circular pin (rod) is inserted in a wall (top view fig. 5a). A rope is wrapped around the end of the pin and is attached to a mass. The mass is lowered with a constant speed and due to friction between rope and pin, heat is produced with a rate P. In figure 5b the side-view is shown. The pin has a length L and a diameter d. For the analysis of this problem we assume that a given amount of frictional heat P (in Watt) is injected homogeneously over the full surface of the end plane $z=L$. This is indicated with the open inclined arrows. The wall has a temperature T_0, and so does the air. Furthermore we assume that the pin at $z=0$ (the boundary position of the wall) has temperature T_0. The pin is made of stainless steel and has a thermal conduction coefficient k_s, the air has a thermal conduction coefficient k_{air}. For a part of this problem there is a heat transfer coefficient h for the heat transfer from the cylindrical pin surface to the air. There is no heat transfer from the end plane at $z=L$ to the air. We use cylinder coordinates (z, r) to describe and analyze the problem.



a). *b).*

Fig. 5 Heated pin due to frictional dissipation at the tip, a). Top view, b). Side view.

(1 pnt) **a).** Formulate the boundary condition for the temperature in this problem. Formulate the boundary condition for the heat flux.

(2 pnt) **b).** First we shall neglect the heat transfer to the air, so $h=0$. Write down a heat balance for a thin segment of the pin between z and $z+\Delta z$. Solve first the heat (flux) equation, and show by using Fourier's law that: $T = Az + B$. Find the values of A and B using the boundary conditions.

(1 pnt) **c).** In order to study a practical case the following data are given: d = 0.02 m, L=0.05 m, the thermal conductivity of stainless steel is k_s=20 W/mK, and P=100 W. Find the temperature difference between the tip of the pin and the wall. What is the temperature difference in case aluminium is used as pin material with a thermal conductivity of k_a=160 W/mK .

(2 pnt) **d).** Now we assume that the heat transfer coefficient h is non-zero, indicating that heat is also transferred to the surrounding air. Set-up the heat balance again for a differential element from z to $z+\Delta z$. Show that:

$$\frac{d^2(T - T_0)}{dz^2} = \beta^2 (T - T_0) ,$$

and specify the value of β.

(3 pnt) **e).** Use the so-called "ansatz" expression e^{kz} for T-T_0 to find the general solution, and then use the boundary conditions to find the full solution of the problem.

In order to determine the effect of the additional heat transfer to the air the value of h can be found from the Nusselt number (Nu) and the following expression for the Nu number is given:

$$Nu = 0.23 \cdot Re^{0.8}$$

The Re number is based on the diameter of the pin (d) and the local wind speed (v_w). The Reynolds and Nusselt numbers are defined as: $Re=v_w d/\nu_{air}$, $Nu=hd/k_{air}$.
Assume that the wind speed is given as $v_w = 5\ m/s$, and the kinematic viscosity of the air is given as $\nu_{air} = 15*10^{-6}\ m^2/s$ and the thermal conduction coefficient of the air is given as $k_{air} = 0.025\ W/mK$.

(2 pnt) **f).** Calculate the value of the Reynolds number, calculate the value of the Nusselt number, and determine the value of the heat transfer coefficient h and finally determine the temperature difference T-T_0 in case a stainless steel pin is used. What is your conclusion comparing the answer with the answer of c).?

END EXAM