Formula Sheet Physical Transport Phenomena (3T320), 2013/2014

Conservation laws of mass, momentum and energy in integral form

(for a control volume V, enclosed by a permeable surface S)

- Mass: $$\frac{\partial}{\partial t}\iiint_V \rho\, dV = -\iint_S \rho(\vec{v}\cdot\vec{n})\, dA$$
- Momentum: $$\frac{\partial}{\partial t}\iiint_V \rho\vec{v}\, dV = -\iint_S \rho\vec{v}(\vec{v}\cdot\vec{n})\, dA + \sum \vec{F}$$
$$\sum \vec{F} = -\iint_S p\vec{n}\, dA + \iint_S (\vec{n}\cdot\vec{\vec{\tau}})\, dA + \iiint_V \rho\vec{g}\, dV + \vec{F}_{other}$$
- Energy: $$\frac{\partial}{\partial t}\iiint_V \rho e\, dV = -\iint_S \rho e(\vec{v}\cdot\vec{n})\, dA + \frac{\delta Q}{dt} - \frac{\delta W}{dt}$$
$$e = \frac{v^2}{2} + gy + u\,;\quad W = W_s + W_p + W_\mu\,;\quad W_\mu = -\iint_S \vec{v}\cdot(\vec{n}\cdot\vec{\vec{\tau}})\, dA$$
$$\frac{\partial}{\partial t}\iiint_V \rho e\, dV = -\iint_S \left(e + \frac{p}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\, dA + \frac{\delta Q}{dt} - \frac{\delta W_s}{dt} - \frac{\delta W_\mu}{dt}$$
$$\frac{V_1^2}{2g} + y_1 + \frac{p_1}{\rho_1 g} = \frac{V_2^2}{2g} + y_2 + \frac{p_2}{\rho_2 g} + \sum h_{turbine} + \sum h_{pump} + h_L \quad \text{(head form)}$$
$$\frac{V_1^2}{2g} + y_1 + \frac{p_1}{\rho g} = \frac{V_2^2}{2g} + y_2 + \frac{p_2}{\rho g} \quad \text{(Bernoulli's equation)}$$

Stress tensor

$$\vec{\vec{\sigma}} = -p\vec{\vec{I}} + \vec{\vec{\tau}}$$

Shear stresses for a laminar Newtonian fluid

- Cartesian coordinates (x, y, z) with $\vec{v} = (v_x, v_y, v_z)$:
$$\tau_{xy} = \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\,;\quad \tau_{yz} = \mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)\,;\quad \tau_{zx} = \mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)$$
- Cylindrical coordinates (r, θ, z) with $\vec{v} = (v_r, v_\theta, v_z)$:
$$\tau_{r\theta} = \mu\left(r\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right)\,;\quad \tau_{\theta z} = \mu\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)\,;\quad \tau_{zr} = \mu\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)$$

Normal stresses for a laminar Newtonian fluid

- Cartesian coordinates (x, y, z) with $\vec{v} = (v_x, v_y, v_z)$:

$$\tau_{xx} = \mu\left(2\frac{\partial v_x}{\partial x} - \frac{2}{3}\vec{\nabla}\cdot\vec{v}\right) \; ; \quad \tau_{yy} = \mu\left(2\frac{\partial v_y}{\partial y} - \frac{2}{3}\vec{\nabla}\cdot\vec{v}\right) \; ; \quad \tau_{zz} = \mu\left(2\frac{\partial v_z}{\partial z} - \frac{2}{3}\vec{\nabla}\cdot\vec{v}\right)$$

- Cylindrical coordinates (r, θ, z) with $\vec{v} = (v_r, v_\theta, v_z)$:

$$\tau_{rr} = \mu\left(2\frac{\partial v_r}{\partial r} - \frac{2}{3}\vec{\nabla}\cdot\vec{v}\right) \; ; \quad \tau_{\theta\theta} = \mu\left(2\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{2}{3}\vec{\nabla}\cdot\vec{v}\right) \; ; \quad \tau_{zz} = \mu\left(2\frac{\partial v_z}{\partial z} - \frac{2}{3}\vec{\nabla}\cdot\vec{v}\right)$$

Conservation laws of mass and momentum in differential form

- Mass: $\dfrac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$ or $\dfrac{D\rho}{Dt} + \rho\nabla\cdot\vec{v} = 0$
- Momentum: $\rho\dfrac{D\vec{v}}{Dt} = -\nabla p + \nabla\cdot\vec{\vec{\tau}} + \rho\vec{g}$

Continuity equation for incompressible flow

- Vector form: $\nabla\cdot\vec{v} = 0$
- Cartesian coordinates (x, y, z) with $\vec{v} = (v_x, v_y, v_z)$:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

- Cylindrical coordinates (r, θ, z) with $\vec{v} = (v_r, v_\theta, v_z)$:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Navier-Stokes equations
(incompressible flow, Newtonian fluid, constant μ)

- Vector form:

$$\rho\frac{D\vec{v}}{Dt} = -\nabla p + \mu\nabla^2\vec{v} + \rho\vec{g}$$

- Cartesian coordinates (x, y, z) with $\vec{v} = (v_x, v_y, v_z)$:

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

- Cylindrical coordinates (r,θ,z) with $\vec{v}=(v_r,v_\theta,v_z)$:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)$$
$$=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)$$
$$=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)$$
$$=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

Rate equations for heat transfer
(with $\dot{Q}=\delta Q/dt$)

- Fourier's law: $\frac{\dot{Q}}{A}=-k\nabla T$
- Newton's 'ansatz': $\frac{\dot{Q}}{A}=h\Delta T$
- Stefan-Boltzmann law: $\frac{\dot{Q}}{A}=\sigma T^4$
- Overall heat transfer: $\frac{\dot{Q}}{A}=\frac{\Delta T}{AR_{\text{thermal}}}=U\Delta T$

Integral form of the energy equation without friction and shaft power

$$\iiint_V \dot{q}\,dV+\dot{Q}=\frac{\partial}{\partial t}\iiint_V \rho e\,dV+\iint_S\left(e+\frac{p}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\,dA$$

Differential form of the energy equation for incompressible media without friction

$$\dot{q}+\nabla\cdot(k\nabla T)=\rho c\frac{DT}{Dt}$$

Dimensionless parameters

$$\text{Re}=\frac{\rho VL}{\mu},\quad \text{Eu}=\frac{p}{\rho V^2},\quad \text{Fr}=\frac{V^2}{gL},\quad C_D=\frac{F/A}{\frac{1}{2}\rho V^2}$$
$$\text{Fo}=\frac{\alpha t}{(V/A)^2},\quad \text{Bi}=\frac{h(V/A)}{k},\quad \text{Pr}=\frac{\nu}{\alpha},\quad \text{Nu}_L=\frac{hL}{k}$$

EINDHOVEN UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF APPLIED PHYSICS
FLUID DYNAMICS LABORATORY

Examination Physical Transport Phenomena (3T320),
Tuesday 28 January 2014, 14.00-17.00 hours.

This example examination is given a maximum of 30 credit points. The maximum number of credit points per subtask is denoted between the brackets.

Problem 1

Answer the following questions with 'yes' or 'no' and give a short argumentation. No credit points will be given for answers without argumentation, even if the answer is right.

(1 pnt) (a) Consider a two-dimensional (dimensionless) velocity field $\vec{v} = (v_x, v_y) = (-ax + ay^2, ay)$, with a a constant. Is this flow incompressible?

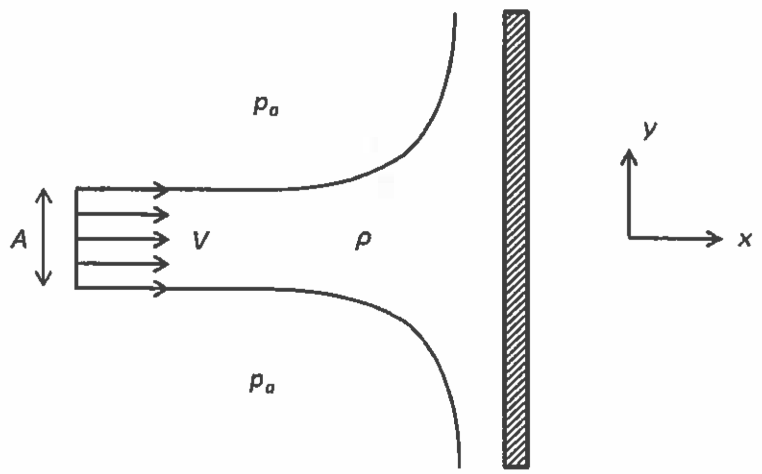
(1 pnt) (b) Is it true that the convective acceleration of a tracer at (x, y) in the flow field defined in (a) is given by $\frac{D\vec{v}}{Dt} = (\frac{Dv_x}{Dt}, \frac{Dv_y}{Dt}) = (a^2x + a^2y^2, a^2y)$?

(1 pnt) (c) Is the viscous stress tensor for the flow field defined in (a) given by

$$2\mu \begin{pmatrix} -a & ay \\ ay & a \end{pmatrix} ?$$

(1 pnt) (d) May the Bernoulli equation be applied in a laminar boundary-layer flow?

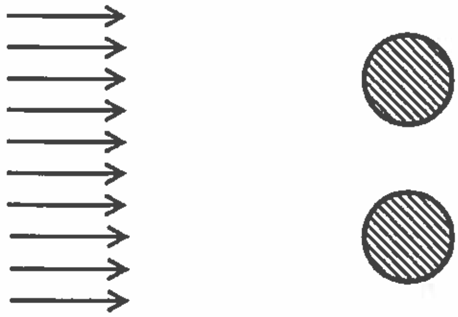
(1 pnt) (e) A steady, incompressible, frictionless, jet of fluid impinges perpendicularly on a flat plate. The jet has a cross-sectional area A, uniform velocity V, uniform density ρ, and is directed in the positive x-direction. The jet is adapted to the ambient pressure p_a. Gravitational forces are to be neglected. Is the force to hold the plate steady given by $\vec{F} = (F_x, F_y, F_z) = (-\rho AV^2, 0, 0)$?



(1 pnt) (f) Consider a nuclear explosion in which a large amount of energy E is suddenly released in the air. Experimental evidence suggests that the distance R travelled by the nuclear-explosion shock wave depends on time t as well as the energy E and the density of the ambient air, ρ. Is it true that this flow problem can be expressed in terms of a single dimensionless parameter

$$\Pi = \frac{R}{E^{1/5} t^{2/5} \rho^{-1/5}} \;?$$

(1 pnt) (g) Consider an inviscid flow around two identical cylinders as sketched below. Is it true that the flow will push the cylinders apart from each other?



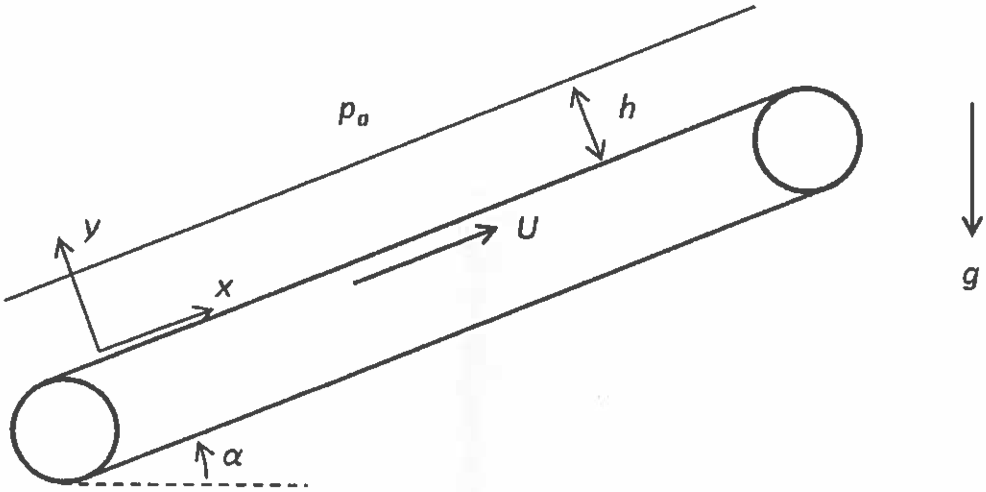
(1 pnt) (h) Do the Biot number and the Nusselt number have the same dimensions?

(1 pnt) (i) We immerse a boiled egg into cold water. The Biot Number is 1. Can we model the temperature-time dependence of the egg according to a lumped-parameter model?

(1 pnt) (j) For laminar flow past a flat surface, the Prandtl Number equals 50. Can Blasius' solution be used to model the thermal boundary layer?

Problem 2

Consider a transport belt that is used to transport a Newtonian incompressible fluid upwards. The transport belt is inclined by an angle α with respect to the horizontal, see sketch below. The upper side of the transport belt moves with a constant velocity U in the direction indicated in the sketch. On top of the belt is a layer of fluid with a constant thickness h. The fluid flow is steady and can be considered fully developed. The length L and width W of the belt are very large, such that boundary effects can be neglected. The flow can be considered two-dimensional (in the plane of the drawing). The Cartesian axes are chosen so that the x-axis is parallel to the belt and the y-axis is normal to the belt. The dynamic viscosity μ and the density ρ are constant. The stress force exerted by the air on the free surface of the fluid layer may be neglected, and the constant pressure of the environment is equal to p_a. The orientation of the gravitational accelleration is indicated in the figure. The components of the velocity in liquid film in the x- and y-direction are named v_x and v_y, respectively.



(1 pnt) (a) State the boundary condition for the velocity at the upper side of the transport belt ($y = 0$).

(1 pnt) (b) Prove that everywhere in the fluid layer the velocity component normal to the transport belt is zero, i.e. $v_y = 0$.

(1 pnt) (c) Show that the boundary condition for the velocity at the interface between fluid and air ($y = h$) is given by $dv_x/dy = 0$.

(2 pnt) (d) Prove that the x- and y-components of the Navier–Stokes equation can be reduced as follows:

$$-\frac{\partial p}{\partial x} - \rho g \sin\alpha + \mu \frac{d^2 v_x}{dy^2} = 0 \ ,$$

$$-\frac{\partial p}{\partial y} - \rho g \cos\alpha = 0 \ .$$

(1 pnt) (e) Use the y-component of the reduced Navier–Stokes equation to determine the pressure as a function of the height y in the fluid layer.

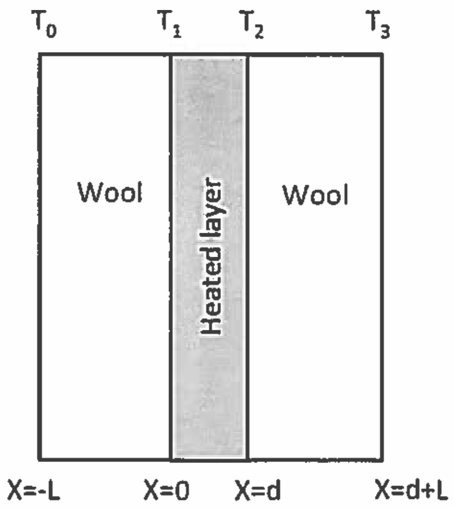
(2 pnt) (f) Determine the velocity profile $v_x(y)$ and make a sketch of the solution.

(1 pnt) (g) Determine the velocity of the belt, U, such that the mass flux in the fluid layer is zero.

(1 pnt) (h) Compute the force that is exerted on the transport belt. Compute also the power needed to keep the belt running.

Problem 3

In a test set-up the thermal properties of electrical blankets are measured by clamping them in between two flat plates with fixed and applied temperatures T_0 and T_3. The locations of these plates are at $x = -L$ and $x = d + L$. The geometry is sketched below. In the center of the blanket an electrically heated layer is present. It has a thickness d and thermal conductivity k_d. The heating per unit of volume is $\dot{q}$. The two wool insulation sections have a thickness L. The thermal conduction coefficient in the wool sections is k_w.



(1 pnt) (a) State Fourier's law explicitly for the three sections of this problem: left wool layer, heated layer, and right wool layer, to compute the heat flux $\dot{Q}/A$ in those layers.

(2 pnt) (b) Assume first that the center layer is *not* heated. Find the relation that describes the heat flux $\dot{Q}/A$ as a function of $T_0 - T_3$. Hint: use the expressions found in (a). Make a graph of the temperature as a function of x for $-L \leq x \leq L + d$. Note that $k_w < k_d$.

(2 pnt) (c) Now in the heated layer an electric current is applied so there is heat generated

with a value $\dot{q}$ per unit volume. The differential form of the steady-state energy equation is given by

$$k\frac{d^2T}{dx^2} + \dot{q} = 0 \ .$$

Integrate this equation twice to find the temperature T in the heated layer as a function of x. Use as boundary conditions that at $x = 0$ (i) there is no heat flux ($dT/dx = 0$) and (ii) $T = T_1$. Compute the heat flux $\dot{Q}/A$ as a function of x in the heated layer.

(2 pnt) (d) Find the expressions for the temperature distribution in the three layers by using the results of part c), and applying the correct boundary conditions. Give explicitly the relationship between $\dot{q}$ and $T_0 - T_3$.

(1 pnt) (e) Make a graph of the temperature as a function of x for $-L \leq x \leq L + d$. Is T_3 higher or lower than T_0?

(2 pnt) (f) Under which condition can the temperature difference $T_1 - T_2$ be neglected with respect to the temperature difference $T_0 - T_3$?